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The Hysteresis of Iron and Steel in a Rotating Magnetic Field

Francis Gibson Baily

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XVIII. *The Hysteresis of Iron and Steel in a Rotating Magnetic Field.*

By Francis Gibson Baily, *M.A., Professor of Electrical Engineering, Heriot-Watt College, Edinburgh.*

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In a paper on Dynamo Electric Machinery by Dr. John Hopkinson ('Phil. Trans.,' 1896), the suggestion is made that the value of the hysteresis of the iron core of a rotating dynamo armature need not be identical with the value obtained when the magnetising force is reversed by passing through a zero value. It was subsequently pointed out by Mr. Swinburne that as a necessary deduction from Professor Ewing's molecular theory of magnetism, the hysteresis of iron in a rotating field, or of iron rotating in a constant field, should show a distinct diminution in value below that in an alternating field, when the magnetic condition of the iron approaches saturation.

According to Ewing's theory hysteresis is due to the formation of stable magnetic combinations between adjacent molecules which tend to resist any movement of the molecular magnets caused by change of direction or magnitude in the magnetising force. On the breaking up of these combinations by such a change in the magnetising force, new arrangements are formed, and the potential energy of position is transformed into kinetic energy of partial rotational movement round the fixed axis of the molecule, which is damped out with or without oscillations above the axis of rotation. It has been further suggested that the damping process may be due to eddy currents induced by the movement of the magnets, but the precise nature of these eddy currents, or the extent to which other retarding influences akin to mechanical friction or viscosity may act has not been determined.

In the alternating field there is no continuity of position of the molecules; the magnetising force passes through a zero value at each alternation, and the molecular combinations may vary in successive alternations, although the average value of the induction and hysteresis remains sensibly constant. Increasing magnetising force and induction result in increased movement of the molecules with increased momentum and consequent dissipation of energy. But in a rotating field of constant value there is no diminution of the strength of the magnetising force. The molecules are always under the same restraint, and the movements impressed upon them will be more uniform and unidirectional. It is true that with small magnetising forces, when but few combinations are dissolved and the movement is of a quasi-elastic nature, the amount of energy dissipated will not be greatly different from that dissipated in an alternating field of equal strength. In fact, since the molecular changes will be forced to take place in one direction only, there will be on the average a greater resistance to movement, and in consequence an increased dissipation of energy. When, however, the field becomes stronger, each molecule will develop a tendency to rotate in synchronism with the field and be less affected by the magnetic influences of surrounding molecules. At this point the hysteresis will show a considerable diminution which will become more marked as the field increases in strength, until finally every molecule will rotate in unison with the field with complete absence of oscillatory movement. The value of the hysteresis under these last conditions will furnish an important clue to the nature of hysteresis. If the hysteresis sensibly vanishes at this point it will be strong evidence that the damping of the movements of the molecules is not due even partially to mechanical friction, but must be produced by some action which is called into play by the rapid oscillations of the molecular magnets, but not by the comparatively slow motion of their rotation with the field. Since the difference between these speeds must be enormous, the damping may be due either to some form of eddy currents or possibly to some form of fluid friction.

The first experimental work upon hysteresis in a rotating field was carried out by Professor FERRARIS ('Atti d. R. Acc. di Torino,' No. 23, 1888). Producing a rotating field by means of two coils at right angles supplied with alternating currents of approximately one quarter difference of phase, he showed that a laminated iron core would rotate by reason of its hysteresis. Beyond proving that at low speeds the hysteresis was independent of the speed, his results were not quantitative, owing to irregularities caused by vibration in the apparatus.

In order to investigate the matter the following apparatus was designed. A powerful electro-magnet is caused to rotate on fixed spindles arranged so that the axis of rotation passes between the pole pieces, which are bored out to a cylindrical form concentric with the axis. A cylindrical armature is held on pivot bearings in the fixed spindles concentrically between the poles. The direction of magnetisation rotates in a plane at right angles to the axis of the armature and concentric with its axis. The armature, though free to rotate in its bearings, is prevented from continuous rotation by a spring attached at one end to its spindle, and at the other end to the fixed spindles of the magnet. Movement of the armature is indicated by a beam of light reflected from a small mirror attached to the armature on to a circular scale concentric with the armature axis.

The apparatus is shown in fig. 1. The electro-magnet is of Swedish iron, 8 sq. centims. in cross-section inside the coils. The pole pieces are bored out to a diameter of 2·3 centims., and subtend an angle of 120°. They are axially of the same length as the armature. The excitation is produced by two coils each of 316 turns of No. 16 wire, one end of the exciting circuit being attached to an insulated ring, the other to the magnet. The current is led in by brush contacts. One of the bearings of the electro-magnet is in the yoke piece, the other is in a gun-metal bracket bolted to the ends of the pole pieces. This bracket carries the insulated collecting ring. A third bearing is arranged in a bracket placed between the pole pieces to prevent any bending of the extended end of the spindle. The spindles are of steel, and are held by set screws in upright gun-metal standards bolted to the bed plate. The rigidity of the electro-magnet was ensured by strong bolts passing through the yoke piece, by the bracket across the ends of the pole piece, and by additional brass brackets of L-section across the sides of the pole pieces. This was found to be quite satisfactory, not the slightest movement of the pole pieces being detected. The armatures were cylindrical in form, and were made of very thin plates strung on a manganese steel spindle 2·8 millims. diameter, and held in place by ebonite washers at each end. Some difficulty was experienced in obtaining satisfactory sheets of charcoal iron, but finally some exceptionally thin plates, ·081 millim. in thickness, were obtained from Messrs. KNIGHT and CROWTHER, who very kindly had them specially rolled for the purpose.

The metal was almost pure iron, containing ·14 per cent. of carbon and only traces of silicon. Its specific resistance was $1{\cdot}2 \times 10^{-5}$ ohm. The plates were insulated from each other by a layer of tissue paper. The armature was 2·60 centims. in length and consisted of 250 plates. The diameter was 1·75 centims. A second armature of hard cold rolled high carbon steel was made with plates ·141 millim. in thickness, consisting of 147 plates similarly insulated with tissue paper. Its specific resistance was $1{\cdot}5 \times 10^{-5}$ ohm.

The springs used were of non-magnetic material in the form of a flat helix. To avoid disturbances the spring was enclosed in an ebonite box through which the armature spindle passed. The armature was supported by the pointed and hardened ends of the steel spindle, which rested on hollow coned bearings of hardened steel, giving sufficient strength while causing a minimum of friction.

The magnet was driven by a small leather belt from an electric motor. Owing to the construction of the apparatus it was impossible to read the speed directly at high speeds, and the value was accordingly calculated from that of the driving motor. The magnetising current was read on a SIEMENS' electro-dynamometer, which had been calibrated from a Kelvin balance.

In order to ascertain whether the presence of windage or eddy currents in the spring attachment or spring or steel spindle produced any appreciable action, a preliminary test was performed with a dummy armature in which the iron was replaced by ebonite, everything else remaining the same. At the highest speeds and with the strongest fields, the effect was inappreciable, and errors from these causes were certainly absent.

When the magnet rotates, the armature revolves with it until the torque exerted by the spring becomes equal to the torque due to the hysteresis and eddy currents in the armature. The rate at which energy is dissipated in the armature is then easily measured, for

Power absorbed = torque × 2π speed of rotation, or
Energy absorbed per revolution = 2π × torque.

Since the restoring force of the spring was found to be proportional to the angle of rotation, the hysteresis and eddy current loss per revolution is proportional to the deflexions of the armature as shown by the reflected beam of light. The reflected beam of light was focussed on to a semicircular translucent scale concentric with, and perpendicular to, the axis of revolution of the electro-magnet. The readings of the movement of the spot of light are, therefore, proportional to the energy absorbed per revolution in the armature.

The springs were calibrated by measuring the deflexion produced by a small weight at the end of a light steel arm attached to the armature. They were calibrated frequently and were found to change only very slightly, even after considerable damage by being wound up, as once or twice happened. The hysteresis per revolution is thus measured directly in terms of the force of gravity acting through a known space.

If l is the length of the arm,
m is the mass of the weight and equivalent weight of the arm,
v is the volume of the iron,
d is the deflexion caused by the weight.

Then a deflexion of 1 division against the spring $= \dfrac{2\pi . l . m . 981}{d . v.}$ ergs per cub. centim. per revolution of energy loss in the armature.

Determination of Eddy Currents.

The only correction of any importance that is required is the loss due to eddy currents. In most tests on hysteresis at a high speed of alternation the eddy current loss has introduced an element of some uncertainty, and therefore great attention was paid to this point. There are two ways in which errors may be caused. Eddy currents will be set up in the armature plates by the movement of the lines of force through them, which will produce a torque on the armature in the same direction as that due to hysteresis. There may also be eddy currents set up in the framework of the apparatus which will modify the magnetic field through the armature, so that it will have a value slightly differing from that produced by the magnetising current alone when the magnet is standing still. This, however, may be shown to be very small. In the rotating parts there will be no eddy currents, since the direction of the lines of force will travel round with the electro-magnet. In the fixed spindles there will be currents set up due to the leakage lines of force which pass through them, which currents will flow along the sides of the spindles and across the ends. The latter portion will be parallel to the lines of force of the field, and hence will produce a magnetic field at right angles to this, causing a slight distortion, but not altering the total magnetic flux between the pole pieces and through the armature. Since the spindles were some distance from the pole pieces, and since no perceptible heating was produced, it may be assumed that no distortion was produced of sufficient magnitude to produce an appreciable change in the value of the hysteresis in different parts of the armature.

The eddy currents in the armature plates may be divided into two portions, (1) those due to the lines of force which pass through the plates parallel to their planes; (2) those which are produced by the fringe of lines of force at the ends of the pole pieces, passing into the armature at the ends in a direction perpendicular to the plane of the plates. The first set affect all the plates equally, and can be calculated with fair accuracy, but the second set only affect the end plates and are not easy to calculate. The distribution of these latter lines of force and currents are shown in fig. 1. It is clear that these lines of force will not penetrate far into the armature, owing to the numerous small air gaps between the plates, and hence the currents will be limited to only a few plates at each end.

Fig. 1.

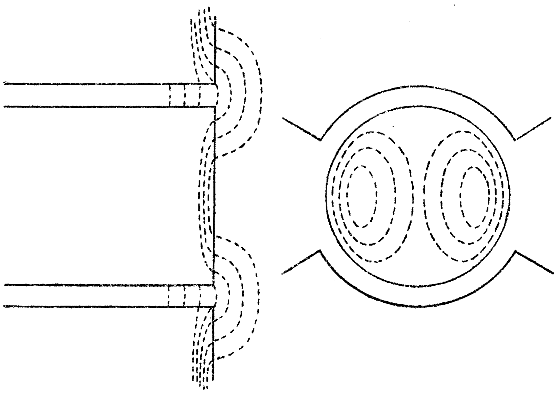
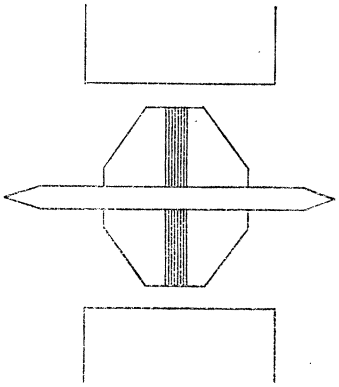


Fig. 2.



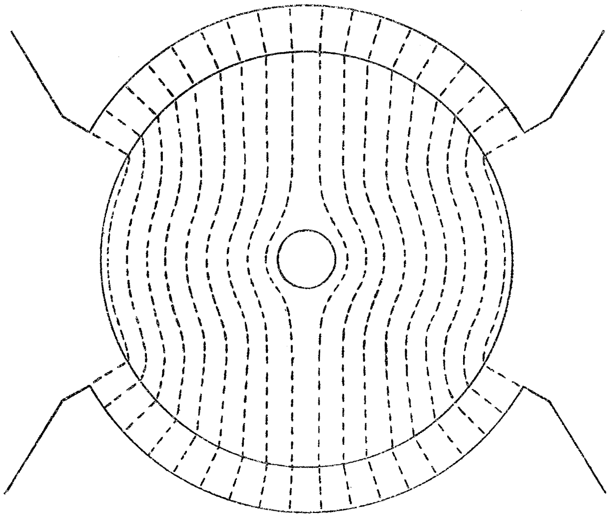
In order to determine the magnitude of this error, an armature was prepared, consisting of only 20 steel plates, insulated with paper as before, and 1·7 centim. diameter (fig. 2). It was placed in the middle of the polar area. If the effect of the eddy currents in the end plates were appreciable, it would be proportionally much increased in this armature, and, at a high increase of speed, there should be an appreciable increase in the deflexion. (It will subsequently be shown that the hysteresis in the steel is independent of the speed.) On experiment, it was found that there was no normal increase in the deflexion. It may therefore be concluded that these end plate eddy currents may be altogether neglected. The test was made with various exciting currents, from a low induction up to the maximum obtainable.

Calculation of Eddy Currents.

As the armature consists of a row of plates insulated from each other, in which the lines of induction lie in the plane of each plate, the problem is reduced to the estimation of the eddy currents induced in a thin plate rotating on a central axis perpendicular to the plane of the plates in a magnetic field, the direction of which is at all points parallel to the plane of the plate. The distribution of the lines of induction in the plate is a matter of some uncertainty, and will vary slightly with different degrees of magnetisation, owing to the variations in the permeability. Since it will be shown, however, that the effect of eddy currents is but small, an approximately accurate distribution will be assumed that will allow of mathematical treatment.

As it has been shown that even at the ends the lines of force do not pass from plate to plate, it will be assumed that their direction is entirely in the plane of the plate.

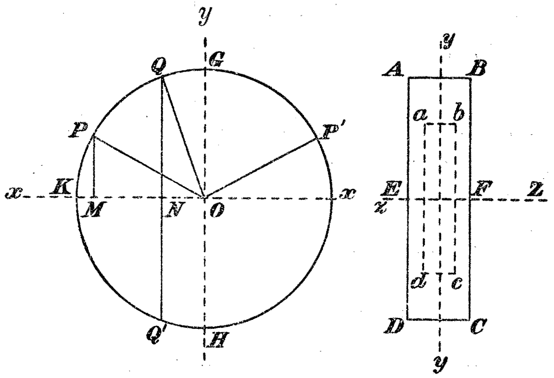
Fig. 3.



Since the magnetic reluctance of the air gap is considerably higher than that of the armature or pole pieces, the distribution of the lines of force in the air gap will be sensibly uniform and radial. It will be assumed that the whole of the lines of force pass in this way to the armature, and the fringe from the sides of the poles will be neglected. This assumption will somewhat increase the value of the eddy currents, as it gives a density in the outer layers slightly in excess of the actual value.

Inside the plate the paths of the lines will be curved, owing to the curve of the circumference and the space occupied by the spindle. Let a distribution be assumed such that the rate of cutting lines of force is proportional to the distance from the centre (neglecting the hole), and such that the portion of each revolution during which any point is cutting lines of force is the same, viz., $\frac{2}{3}$ of each revolution, and that the rate of cutting lines of force during that time is uniform. For the remaining $\frac{1}{3}$ revolution, the point is travelling along the lines of force. A reference to the fig. 3, illustrating this distribution, will show that such a distribution is very approximately the actual condition.

Fig. 4.



Let QP Q′P′ be a plate in the armature, fig. 4, and let ABCD be a section of this plate at some point QQ′ between G and P, this part being covered by the pole piece and PK being between the pole pieces.

Let the thickness of the plate AB $= 2d$, and let the diameter GH $= 2md$, and let the angle subtended by the pole piece $= 120°$. In the samples used, m has a value of 150 to 200.

In any section QQ′ the path of the current induced in the plate by its revolution in a magnetic field parallel to the plane of the plate, will be approximately rectangular and similar to ABCD.

Let ON $= x$, and let $abcd$ be the path of the current through a point in OF at a distance z from O.

Then the length of the path

$$= 4z + 4mz\frac{\mathrm{QN}}{\mathrm{QO}} = 4z\left(1 + m\frac{\sqrt{(m^2d^2 - x^2)}}{md}\right).$$

Let the thickness of the element be dx, and the breadth along $bc = dz$. Then breadth along $ab = mdz$. Let $\rho =$ specific resistance of the metal in ohms.

Then the resistance of element

$$= \frac{4z\rho}{dz\,.\,dx}\left(\frac{1}{m} + m\frac{\sqrt{(m^2d^2 - x^2)}}{md}\right) = \frac{4z\rho}{m\,.\,d\,.\,dz\,.\,dx}\left(d + m\sqrt{m^2d^2 - x^2}\right).$$

Let the induction across diameter of plate $=$ B. Then the induction in the air gap $= \mathrm{B}\frac{3}{\pi}$ assuming a distribution of magnetisation as above.

And according to the previous assumption the induction at right angles to the tangent at any point in the path as far as the line OP will be also $= \frac{3\mathrm{B}}{\pi}$.

Let n = number of revolutions per second of the armature.

Then EMF in the circuit $= \frac{3B}{\pi} . 2\pi nmz . 4z.$

$= 24\ Bnmz^2\ 10^{-8}$ volts.

Then the energy spent in eddy currents per second (from OP′ to OP) in the elementary circuit $abcd . dx . dz$

$$= \frac{E^2}{R} = \frac{24^2\ B^2 . n^2 . m^2 . z^4 . 10^{-16}\ m . d . dz . dx}{4\rho z\ (d + m\ \sqrt{m^2d^2 - x^2})} = \frac{144\ B^2 . n^2 . m^3 . z^3 . d . dz . dx}{10^{16}\rho\ (d + m\ \sqrt{m^2d^2 - x^2})}.$$

Therefore the loss in the whole section Q Q′

$$= \frac{144B^2n^2m^3\ d}{10^{16}\rho} . \frac{dx}{d + m\sqrt{m^2\ d^2 - x^2}} . \int_0^d z^3\ dz,$$

$$= \frac{36B^2n^2m^3\ d^5}{10^{16}\rho} . \frac{dx}{d + m\sqrt{m^2\ d^2 - x^2}} .$$

Over the whole plate the loss is obtained by taking sections from G to P; *i.e.*, integrating from $x = 0$ to $x = \mathrm{OM} = \mathrm{OP} \sin 60° = md \sin \pi/3$.

The integral $\int_0^{m\,d \sin \pi/3} \frac{dx}{d + m\sqrt{m^2\ d^2 - x^2}}$ may be solved by writing $x = m\,d \sin \vartheta$, when it becomes $\int_0^{\pi/3} \frac{m \cos \vartheta}{1 + m^2 \cos \vartheta}\, d\,\vartheta$, but since m is a large number and d is small, it may be simplified to the form $\frac{1}{m} \int \frac{dx}{\sqrt{m^2\ d^2 - x^2}}$.

This

$$= \frac{1}{m} \left[\sin^{-1} \frac{x}{m\ d} \right]_0^{m\,d \sin \pi/3} = \frac{\pi}{3m}.$$

Therefore the eddy current losses in each plate

$$= 2 . \frac{36B^2n^2m^3\ d^5}{10^{16}\rho} . \frac{\pi}{3m} \text{ watts.}$$

The volume of each plate $= 2\pi m^2\ d^3$, therefore the eddy current loss per cubic centimetre

$$= \frac{12B^2n^2\ d^2}{10^{16}\rho} \text{ watts.}$$

For calculation it is more convenient to replace d the half thickness of the plate by t the thickness. Then the loss per cub. centim. $= \frac{3B^2n^2t^2}{10^{16}\rho}$. The expression given by Professor J. J. Thomson for the loss due to eddy currents in iron plates in an alternating magnetic field $= 1 \cdot 67 \frac{B^2 n^2 t^2}{10^{16} \rho}$, the difference in the constant being due to the fact that the distribution of E.M.F. is different in the two cases, the average being larger for the rotating field, and also that in the latter the average length of path and resistance of the eddy current circuits are smaller.

It is more convenient to estimate the loss due to eddy currents in ergs per revolution. This $= \frac{3B^2 n t^2}{10^9 \rho}$ ergs per cub. centim. per revolution.

For the soft iron $t = \cdot 0081$ and $\rho = 1 \cdot 2 \times 10^{-5}$; $\therefore$ eddy losses $= 1 \cdot 64 \times 10^{-8} B^2 n$.
For the hard steel $t = \cdot 0141$ and $\rho = 1 \cdot 5 \times 10^{-5}$; $\therefore$ eddy losses $= 4 \cdot 0 \times 10^{-8} B^2 n$.

Determination of Induction in Armature.

The induction in the armature was measured by winding a few turns of wire round the iron, through holes in the ebonite washers. The electro-magnet was excited by a measured current. The coil was placed in series with a ballistic galvanometer, and the throw observed when the coil was suddenly moved half round. By this means all effects of the stray field of the magnet on the galvanometer and leading-in wires were avoided. These were, however, very small, since the galvanometer was 30 feet distant, and the connecting wires were carefully stranded and insulated.

The galvanometer was standardised by a Clark's standard cell and a Muirhead standard condenser. The resistance of the galvanometer circuit and coil was made large, to avoid damping due to currents induced in the coils by the swing of the needle. Other causes of damping being the same both for the induction throw and the condenser throw, it was not necessary to allow for this. In the soft iron armature, and to a less extent in the steel armature, the paper occupies some space, and with a strong field the lines of force through the paper are an appreciable quantity. The strength of the field was determined by winding an air coil round the ebonite washers at the end, and measuring the throw. This gives a fairly accurate value for the strength of the field, as it is just outside the iron. The area occupied by paper and air in the armature was calculated, and the proportional correction subtracted from the total number of lines of force as given by the coil round the iron. It did not amount to more than 5 per cent. in the strongest field. The results of the calibrations are given in the Diagrams 2 and 3, from which the values for the induction in the tables of readings are taken.

In the value of the area of the iron, the small hole in the centre was subtracted from the total cross-section of the armature, so that over a part of the armature the value of B will be really somewhat *less* than that given. But for the part under the pole-pieces the value will be nearly correct.

Test of Permeability and Hysteresis in a Reversing Field.

The method adopted was that used by EWING in his more recent experiments and described by him in his paper ('Phil. Trans.,' 1894), in which a continuous ring of iron is wound uniformly round with a magnetising coil, and a secondary coil is connected to a ballistic galvanometer. The current is always brought to the maximum value, and the throw of the galvanometer taken when the current is reduced (counting the sign algebraically), until the value of the reversed initial current is reached. Each change being practically instantaneous, it is probable that this method gives much the same value for the hysteresis as a rapidly alternating current does, but the point has not been experimentally proved. Dr. JOHN HOPKINSON's tests* on hardened steel show close similarity in the permeability with an alternating current and with a step-by-step method, but his experiments are not conclusive for the hysteresis value. As the iron was well laminated, there would be, according to EWING, very little creeping effect in the magnetisation of the ring. For this purpose rings were made of the iron and steel, consisting of a narrow strip wound in several layers, the samples being taken from portions of the sheets of material used for making the armatures, and the physical conditions being kept as nearly as possible unchanged. Primary and secondary windings were wound on, the secondary being of fine wire placed immediately on the iron, so that no correction for air space was necessary. The dimensions of the rings were :—

Soft iron.—Area of metal . . . . . = ·0730 sq. centim.
Mean circumference of ring = 20·5 centims.

Steel.—Area of metal . . . . . = ·0366 sq. centim.
Mean circumference . . . = 22·8 centims.

The method being well known requires no further description.

The hysteresis is given by the expression $\frac{1}{4\pi}\int \mathrm{H}d\mathrm{B}$, where $\int \mathrm{H}d\mathrm{B}$ is the area of the curve. B and H being in absolute measure, the hysteresis is expressed in ergs per cubic centimetre per cycle.

By measuring the areas of the closed curves and dividing by 4π the following values were obtained :—

	B =	Hysteresis =	
Soft Iron.	3,060	1,280	ergs.
	6,850	4,140	,,
	10,300	7,450	,,
	11,800	9,100	,,
	14,100	12,600	,,

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* 'Electrician,' September, 1892, and May, 1893.

	B	Hysteresis (ergs)
Hard Steel.	2,720	3,580
	6,600	13,850
	9,800	24,000
	11,900	33,300
	14,400	68,000

These values are shown in the curves on Diagrams 4 and 5.

Test of Variation of Hysteresis with Induction in Rotating Field.

The following tests were taken to determine the nature of the variation of hysteresis as the induction is increased. Both hard cold-rolled steel, and soft charcoal iron were used.

The machine was kept running continuously through the test, and the current was kept on continuously and gradually increased, except when very heavy currents were used. Some 10 or 15 secs. were allowed to pass before the reading was taken to give the deflection time to become steady. The effect of not complying with these conditions will be discussed later.

The shape of the pole pieces was varied twice to obtain a stronger field, and the armature was reduced in length. The first change was to cut away the poles, making the length of the polar area 1·8 centim. instead of 2·5 centims., and later coned ends were fitted to give a stronger field, the latter shape being shown in the Diagram (1).

The calibration curves for the three shapes are shown in Diagram 2.

The hard steel armature was used.

Readings were taken at 28 revolutions and at 56 revolutions per second, the agreement being fairly close.

With a shortened armature and stronger field, a set was taken at 34 revolutions per second.

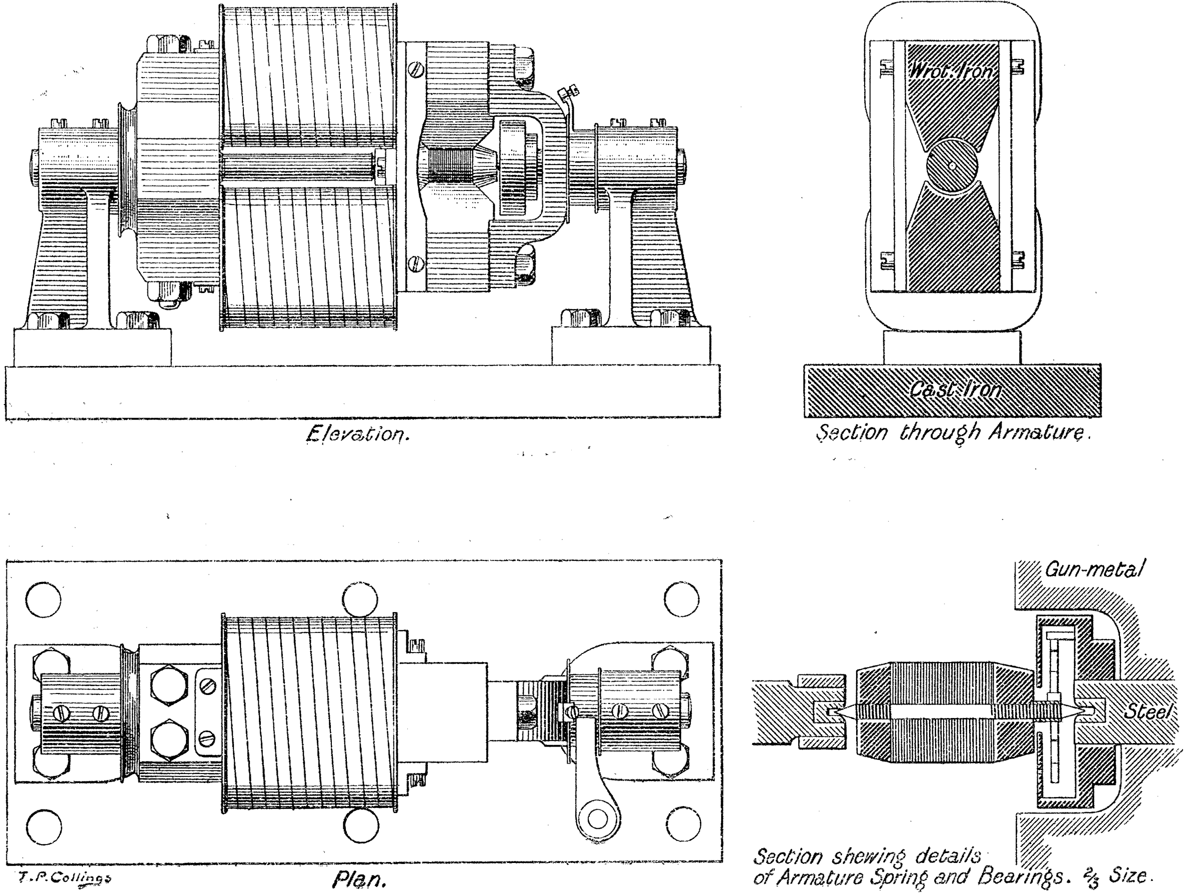
With the shortened armature and cone-shaped pole pieces a set was taken at 26 revolutions per second. This was the strongest field that could be obtained.

The results are given in the adjoining table, and the curves are plotted in Diagram 4, together with the hysteresis curve, by the method of reversals for a piece of the same sample of steel.

The whole curve is shown, from very low values of B up to nearly 20,000. It will be seen that all four sets of readings are very fairly concordant. A maximum value of 44,000 ergs is reached at an induction which varies between 14,000 and 15,000.

At the beginning the curve rises very slowly and then turns up sharply along an almost straight line. Between 9,000 and 10,000 there is a slight flexure in all of them and another straight piece until the maximum is reached. Here there is an abrupt bend and a more rapid descent on the other side, again almost straight, the

Diagram 1.



Hysteresis Measuring Apparatus.

Diagram 2.

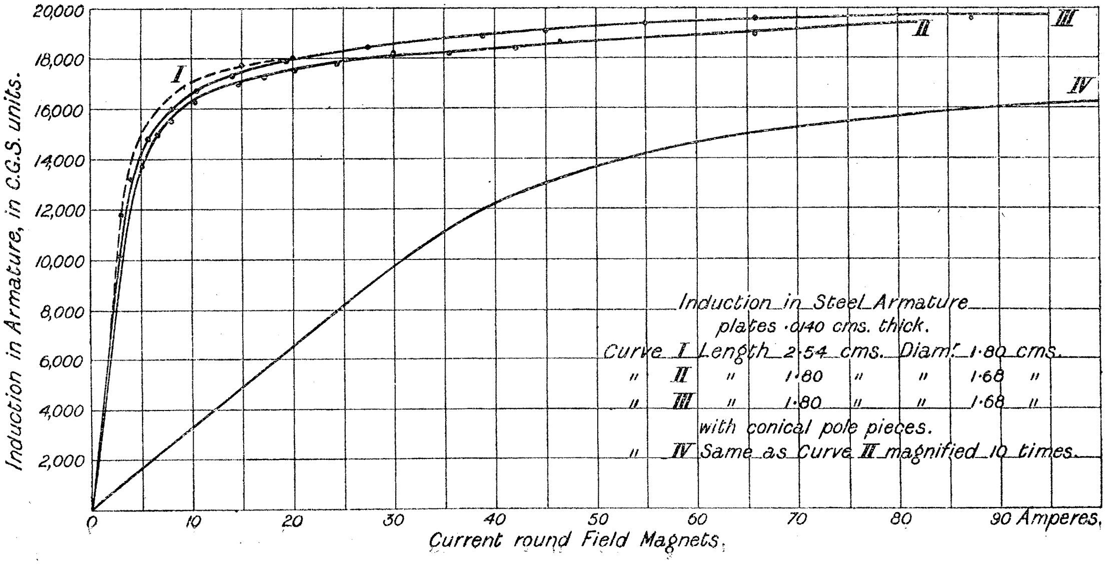


Diagram 3.

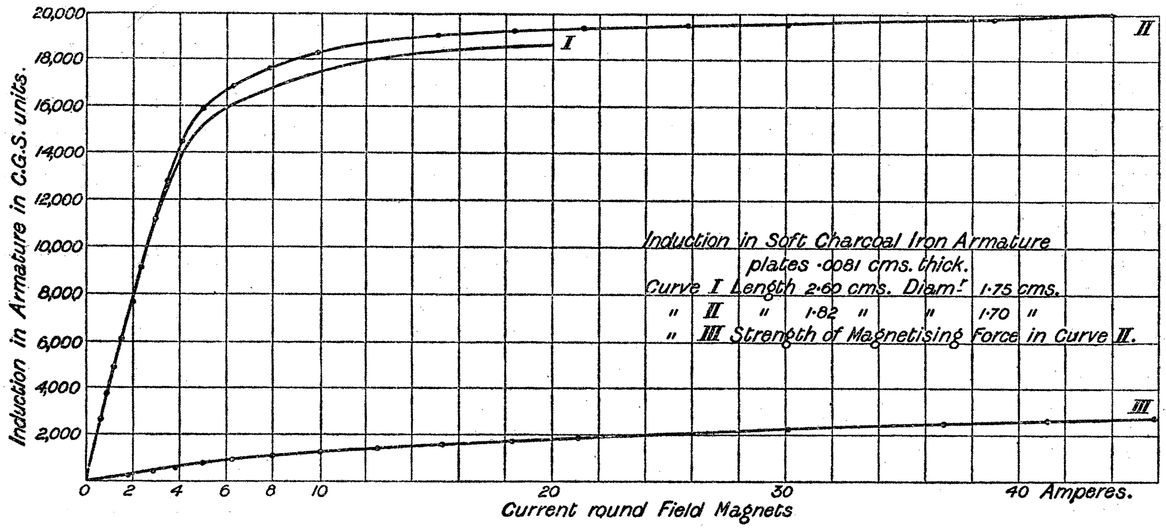
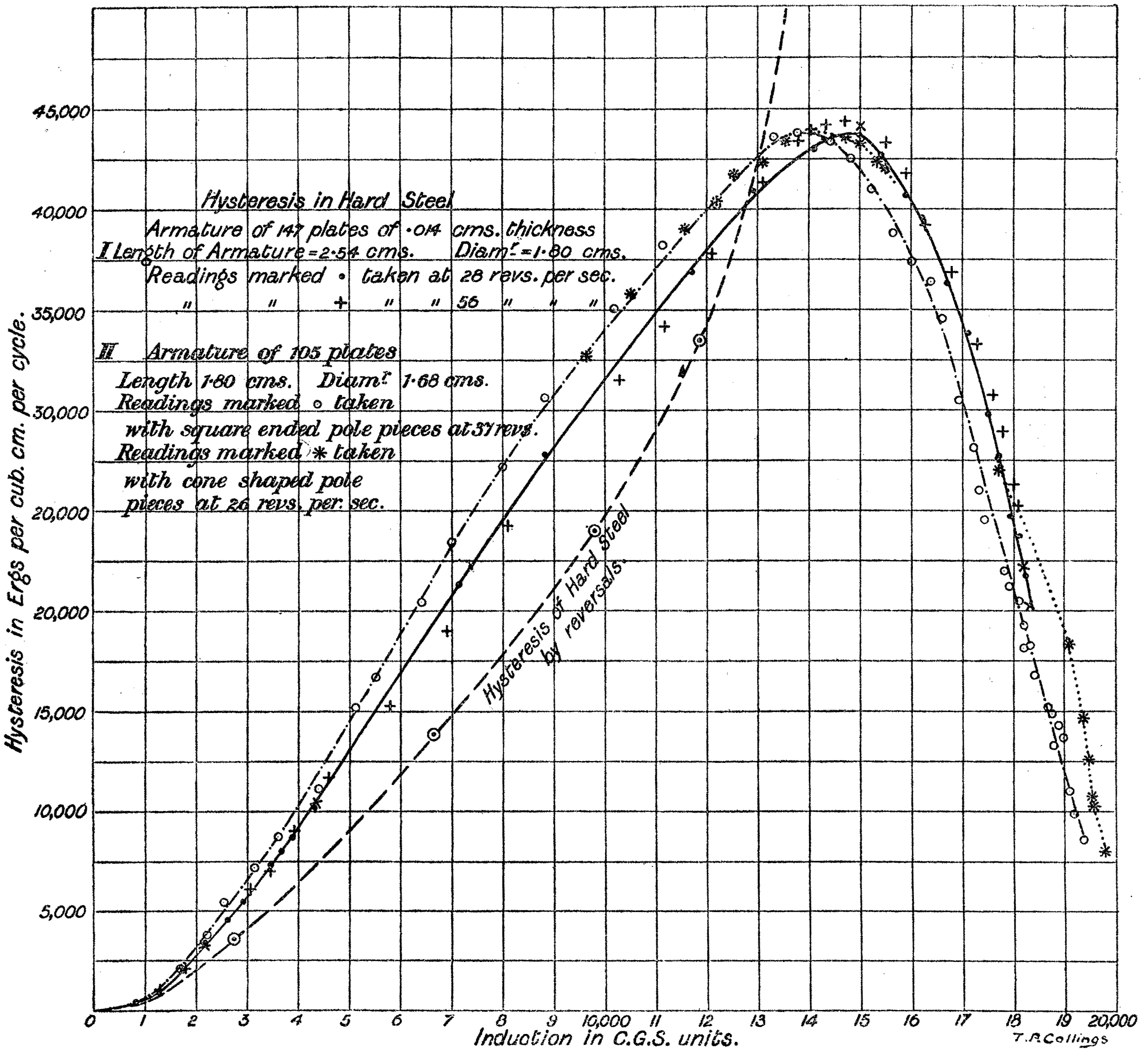


Diagram 4.



lowest value, 8,000 ergs, less than one-fifth of the maximum, being reached without any sign of another flexure.

This corresponds perfectly to the three stages in the B/H curve of hard steel, the rapid rise occurring during the rapid rise of the induction, and the maximum value being reached just at the bend before the approach to saturation. As saturation is more nearly reached, the hysteresis becomes rapidly smaller, the curve showing no sign of bending off asymptotically.

The values given in the tables for hysteresis are corrected for eddy currents, the magnitude of the correction being given at the foot. The maximum value of the correction at 56 revolutions per second, when B = 18,400 and the hysteresis is small, is only 4 per cent. of the hysteresis, and about the same for the smaller value of the hysteresis in the stronger field at the slower speed.

It will be noted that the ballistic curve lies considerably below the rotating field curve up to a point close to the maximum, after which it rises far above the other. The meaning of this will be discussed later.

TABLE I.—Hard Steel Armature of 105 Plates of thickness ·0141 centim. Diagram 4. Square pole pieces. Speed, 37 revs. per sec.

B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.
1,700	2,100	14,800	42,500	18,900	14,200
2,200	3,600	15,200	41,000	19,000	13,800
2,520	5,500	15,600	38,800	19,100	10,900
3,100	7,200	16,000	37,400		
3,600	8,850	16,400	36,500	18,200	18,100
4,400	12,300	16,600	34,600	18,400	16,700
5,100	15,200	16,900	30,500	18,700	15,100
5,500	16,800	17,200	28,100	18,800	13,200
6,400	20,500	17,300	26,000	19,000	10,400
7,000	23,600	17,400	24,500	19,200	9,860
8,000	27,300	17,500	24,300	19,400	8,580
8,800	30,800	17,800	22,000		
10,200	35,100	17,900	21,200		
11,100	38,300	18,100	20,500		
12,200	42,000	18,200	19,300	Eddy currents = 1·5 × 10^{-6} B^2 ergs per cub. centim. per rev.	
13,300	43,700	18,300	18,200		
13,800	43,800	18,500	16,700		
14,400	43,300	18,800	14,900		

TABLE II.—Hard Steel Armature of 147 Plates of thickness ·0141 centim. Variation of Hysteresis with induction. Diagram 4.

Speed, 28 revs. per sec.				Speed, 56 revs. per sec.			
B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.
865	400	16,200	39,600	1,300	1,000	14,700	44,400
1,300	950	16,700	36,300	1,780	2,010	15,000	44,100
1,730	2,010	17,100	33,800	2,160	3,230	15,500	43,300
2,160	3,420	17,500	29,800	2,610	4,520	15,900	41,800
2,610	4,580	17,700	27,700	3,040	6,040	16,300	39,200
2,940	5,500	17,900	24,700	3,460	7,050	16,800	36,900
3,450	7,300	18,100	23,700	3,900	9,050	17,300	33,400
3,680	7,800	18,200	21,700	4,320	10,400	17,600	30,800
3,900	8,700			4,600	11,750	17,800	29,000
4,320	10,200	Eddy currents = 1·1 × 10⁻⁶ B² ergs per cub. centim. per rev.		5,800	15,200	18,000	27,600
7,160	21,400			6,900	18,900	18,100	25,300
8,850	27,800			8,100	24,200	18,200	22,200
11,700	36,900			10,300	31,400	18,300	20,300
12,900	40,800			11,200	34,100		
14,100	43,000			12,100	37,800	Eddy currents = 2·2 × 10⁻⁶ B² ergs per cub. centim. per rev.	
14,300	43,500			13,100	41,300		
15,000	43,400			13,800	43,500		
15,400	42,700			14,000	43,900		
15,900	40,700			14,300	44,200		

TABLE III.—Hard Steel Armature of 105 Plates of thickness ·0141 centim. Diagram 4. Coned pole pieces. Speed, 26 revs. per sec.

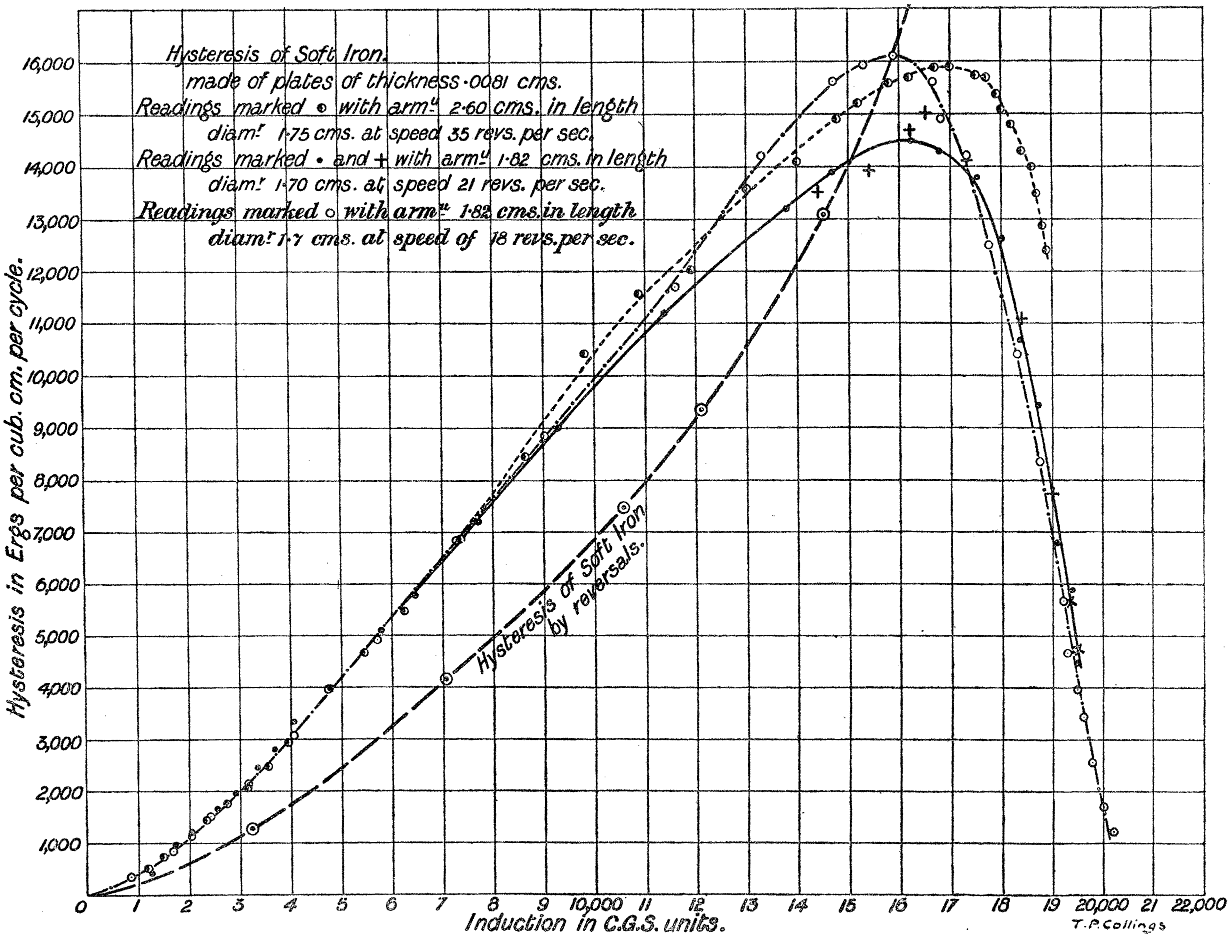
B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.
9,700	32,700	15,100	43,300	19,650	10,700
10,600	36,300	15,400	42,400	19,700	10,200
11,600	39,100	15,600	42,000	19,800	7,900
12,200	40,700	17,800	29,500		
12,600	41,800	18,200	24,700		
13,200	42,300	19,200	18,300	Eddy currents = 1·0 × 10⁻⁶ B² ergs per cub. centim. per rev.	
13,600	43,600	19,500	14,800		
14,800	43,600	19,600	12,500		

Hysteresis of Soft Charcoal Iron.

The steel armature was replaced by the soft iron armature of 250 plates, ·0081 centim. thick. This was subsequently reduced to 156 plates, the calibration curves being given in Diagram 3. Curve 2 was carried further in later experiments, the maximum induction attained being 20,200.

Several sets of readings were taken at different speeds with both sizes of armature, some of the results being shown in Diagram 5, together with the ballistic curve. The general shape of the curve is similar to that of the steel, the three stages being clearly marked—the slow beginning, then a long straight rise, with a less distinctly shown flexure in the middle, and a sharply defined maximum, followed by a sudden drop. The maximum occurs at a higher induction than in the steel, and the drop afterwards is more rapid.

Diagram 5.



It may be noted that, while the first and last parts of the curve are in good agreement, there is considerable divergence near the maximum, and repeated experiment failed to remove this. In fact, when kept running at the maximum value, the deflection was by no means steady, showing that it is a genuine phenomenon, and not due to errors in the apparatus. Occasionally, in the latter half of the curve, the same phenomenon occurred, the value altering slightly for a few readings, and then coming back on to the original curve, without any alteration or stoppage of the apparatus.

In these experiments the induction was pushed as high as possible, and a consistent continuous curve was obtained down to a very low value of the hysteresis. The minimum value obtained at an induction of 20,200 is only one-thirteenth of the maximum value, and the curve shows no signs of turning off again. It is therefore highly probable that the hysteresis vanishes altogether at a slightly higher induction, although the saturation point has hardly been reached. These last readings are very difficult to obtain, as the smallest irregularity or incidental error entirely vitiates them, and for some time the results were not good. However, the curve given was obtained several times, and the points marked are the mean of several readings, all in good agreement, so that it may be taken as correct.

The comparison with the ballistic curve exhibits the same features as in the case of hard steel, the relation between the two curves being singularly alike. It may therefore be concluded that the positions will be the same for other samples of iron and steel, since these two occupy extreme positions among the various types.

That the hysteresis in a rotating armature at low induction should be greater than in an alternating field is quite intelligible, since the movement is more gradual, and is free from sudden shocks. There is also not so much choice in the direction of movement, and hence some of the molecular combinations will offer more resistance to dissociation. The point is of considerable importance in the design of large dynamo armatures, which are usually worked at an induction of about 10,000 or 8,000 C.G.S. At this part of the curve the value of the hysteresis is some 50 per cent. higher than that given by a ballistic test, and allowance must be made accordingly for the larger amount of heating. On the other hand, in small ring armatures, which are worked at a high induction, the hysteresis will be considerably lower than the value given by the ballistic method.

Diagram 6.

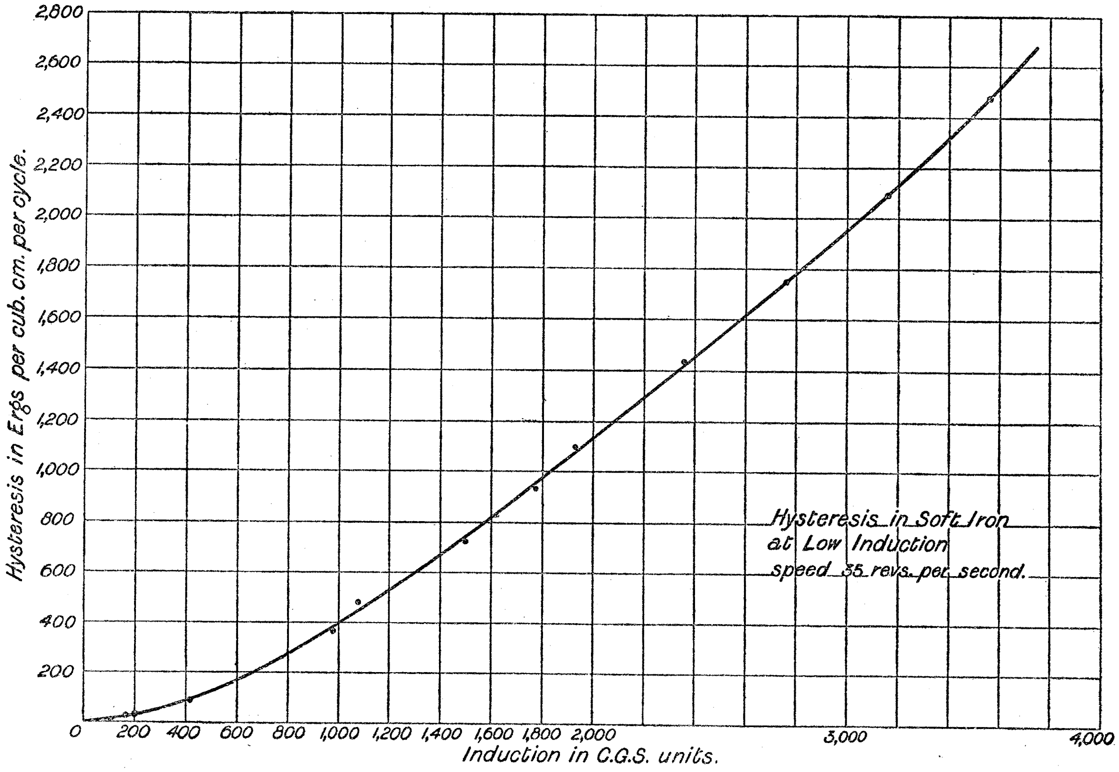


Diagram 7.

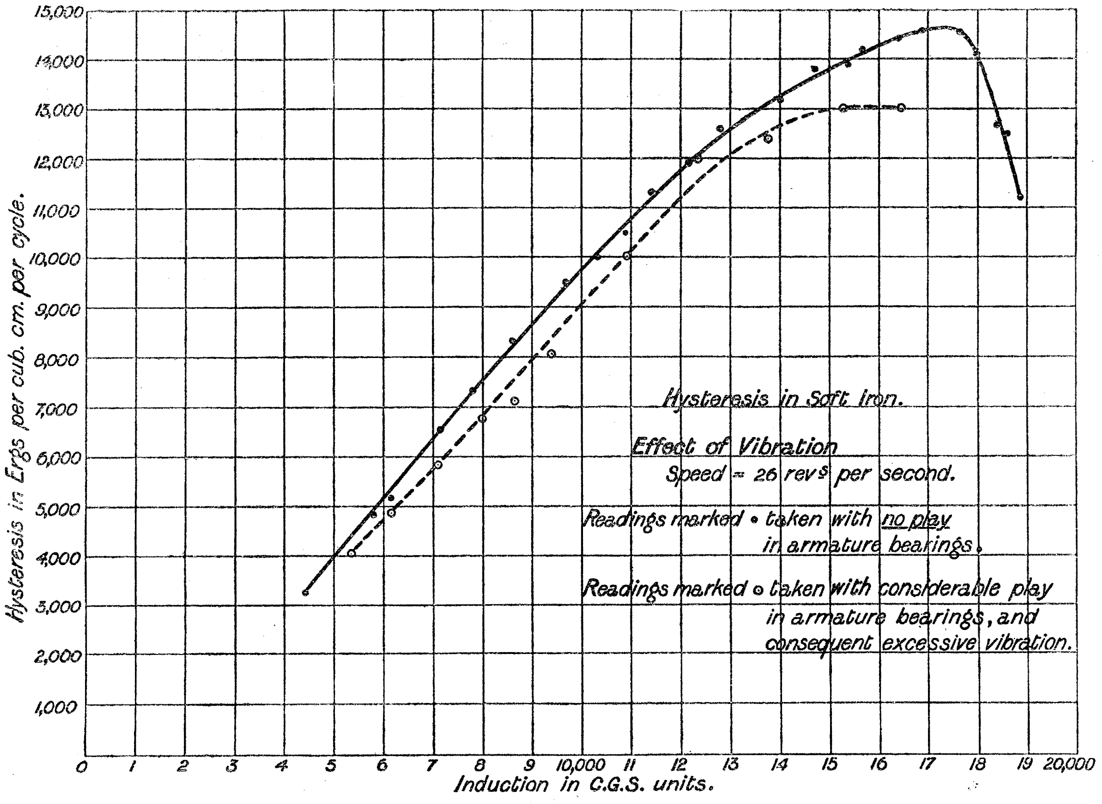


Diagram 8.

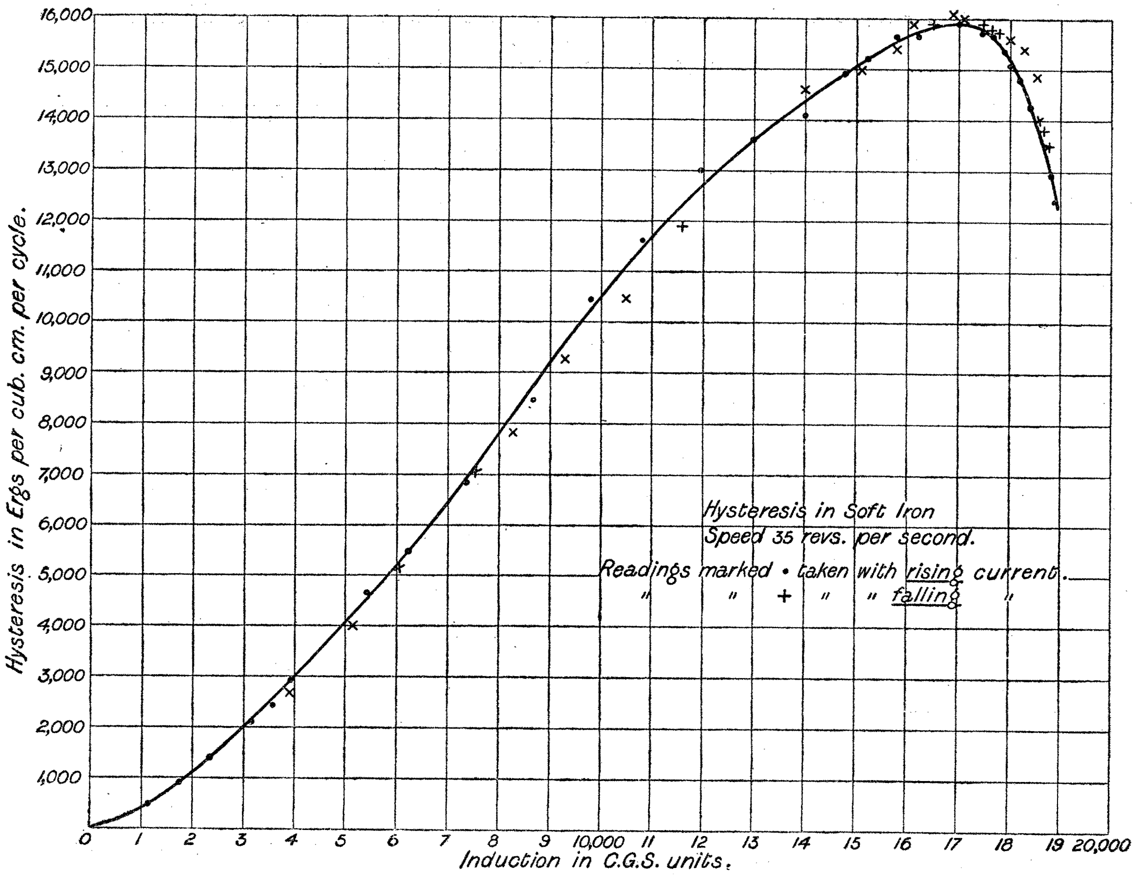
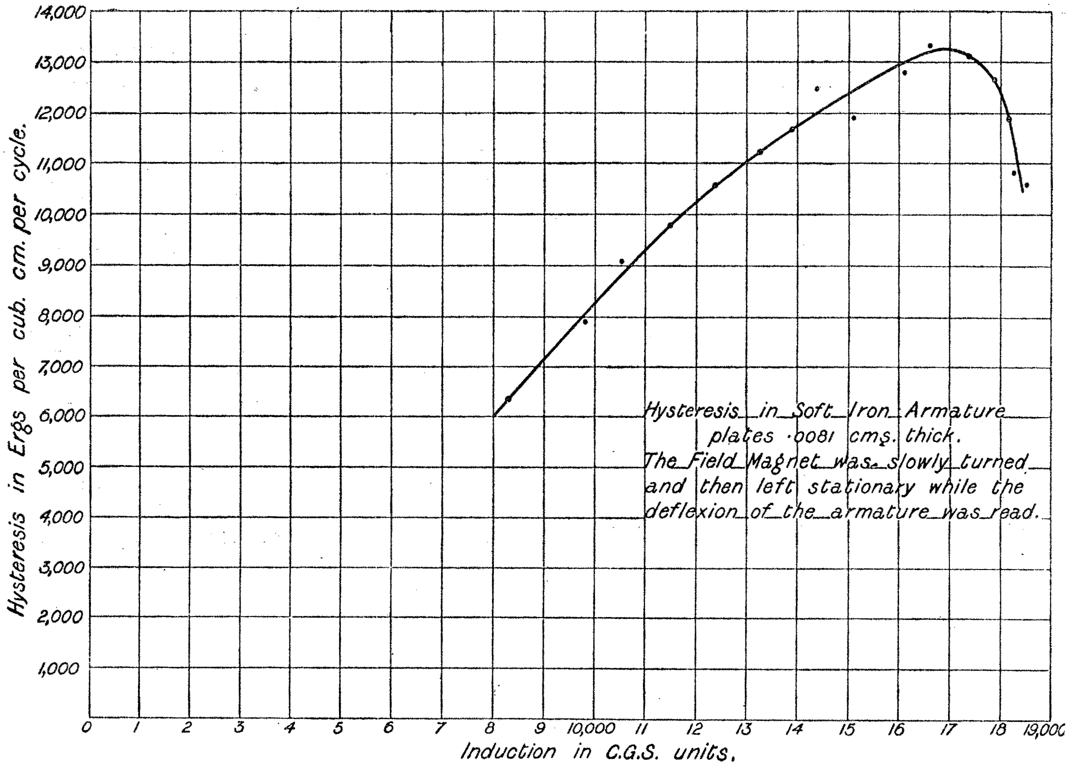


Diagram 9.



PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

TABLE IV.—Soft Iron Armature of 250 Plates of thickness ·0081 centim. Diagram 5. Speed, 35 revs. per. sec.

B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.
102	9·6	5,420	4,650	18,200	14,800
166	22	6,250	5,430	18,400	14,300
193	26	7,360	6,850	18,600	14,000
410	87	8,640	8,450	18,700	13,500
593	97	9,800	10,450	18,800	12,900
980	367	10,850	11,600	18,900	12,400
1,180	483	11,950	13,000		
1,500	725	13,000	13,600		
1,770	933	14,000	14,100		
1,930	1,090	14,800	14,900		
2,360	1,430	15,200	15,200		
2,760	1,750	15,800	15,600		
3,160	2,090	16,200	15,700	Eddies = $6\cdot0 \times 10^{-7}$ B^2.	
3,560	2,480	17,000	15,900		
3,940	2,960	17,500	15,750		
		17,700	15,700	Also plotted on Diagram 8.	
		17,900	15,400		
Also plotted on Diagram 6.		18,000	15,100		

TABLE V.—Hysteresis in Soft Iron Armature of 156 Plates of thickness ·0081 centim. Square pole pieces. Diagram 5. Speed, 21 revs. per sec.

B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.
1,290	390	13,800	13,200	16,200	14,700
1,660	1,000	14,700	13,900	16,500	15,000
2,020	1,160	16,200	14,500	17,300	14,200
2,580	1,630	16,800	14,300	18,000	12,600
2,940	1,940	17,500	13,800	18,400	11,100
3,310	2,480	18,000	12,600	19,000	8,800
3,680	2,800	18,400	12,700	19,100	6,800
4,050	3,340	18,700	9,500	19,350	5,700
4,150	3,410	19,100	6,800	19,500	4,800
4,780	3,950	19,400	5,900		
5,800	5,100	19,500	4,500		
6,450	5,800				
7,700	7,200				
9,300	9,000	14,400	13,500	Eddy currents = $3\cdot5 \times 10^{-7}$ B^2 ergs per cub. centim. per rev.	
10,400	11,200	15,400	13,850		

TABLE VI.—Hysteresis of Soft Iron Armature with 156 Plates of thickness ·0081 centim. Speed, 18 revs. per sec. Diagram 5.

B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.
830	350	11,600	11,700	19,200	5,700
1,200	545	13,300	14,200	19,300	4,600
1,660	816	14,700	15,600	19,500	3,900
2,020	1,110	15,300	15,900	19,600	3,400
2,400	1,500	15,900	16,100	19,800	2,600
3,130	2,180	16,600	15,600	20,000	1,700
4,050	3,060	16,800	14,900	20,200	1,200
4,700	3,940	17,300	14,200	Eddy currents = $3\cdot0 \times 10^{-7} B^2$.	
5,700	4,900	17,800	12,500		
7,260	6,800	18,300	10,400		
9,020	8,850	18,800	8,400		

TABLE VII.—Soft Iron Armature of 250 Plates of thickness ·0081 centim. Variation of Hysteresis with Excessive Vibration. Diagram 7. Speed, 26 revs. per sec.

Armature almost tight in bearings.				Armature very loose in bearings. Excessive vibration.	
B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.
4,420	3,280	15,400	13,900	5,340	4,050
5,700	4,840	15,700	14,200	6,160	4,820
6,160	5,200	16,400	14,400	7,100	5,800
7,170	6,550	16,900	14,600	8,000	6,750
7,820	7,330	17,700	14,600	8,650	7,100
8,650	8,300	18,000	14,100	9,400	8,050
9,700	9,500	18,400	12,700	10,900	10,000
10,300	10,000	18,600	12,500	12,300	12,000
10,900	10,500	18,900	11,200	13,800	12,400
11,400	11,300			15,300	13,000
12,200	11,900			16,500	13,000
12,800	12,600	Eddy currents = $4\cdot3 \times 10^{-7} B^2$.			
14,000	13,200				
14,700	13,800				

Higher values of B were not possible as the vibrations threatened to break the pivots of the armature.

Table VIII.—Soft Iron Armature of 250 Plates of thickness ·0081 centim. Speed, 35 revs. per second. Diagram 8. Starting with maximum current and decreasing.

B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.
18,800	13,500	17,100	16,000	9,300	9,250
18,700	13,800	16,900	16,100	8,300	7,800
18,600	14,000	16,500	15,900	7,550	7,050
18,500	14,900	16,100	15,900	6,070	5,120
18,300	15,400	15,800	15,400	5,150	4,000
18,000	15,600	15,100	15,000	3,960	2,680
17,800	15,750	14,000	14,600		
17,600	15,800	11,600	11,900		
17,500	15,900	10,500	10,450	Eddy currents = $6{\cdot}0 \times 10^{-7}$ B^2.	

Table IX.—Hysteresis and Induction in Soft Iron Armature of 250 Plates of thickness ·0081 centim. The Armature slowly turned and then left at rest and the reading taken. Diagram 9.

B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.	B.	Hyst. in ergs per cub. centim. per rev.
8,300	6,370	13,300	11,200	16,600	13,300
9,800	7,900	13,900	11,700	17,400	12,700
10,500	9,100	14,400	12,500	18,200	11,900
11,500	9,800	15,100	11,900	18,300	10,800
12,400	10,600	16,100	12,800	18,500	10,600

In Diagram 6 is given the first part of the curve plotted on a magnified scale, and determined by means of a very weak spring, so that the deflexions were large. The initial values are small, but the increase is rapid up to an induction of 1,400, after which the curve becomes more nearly straight.

The effect of violent vibration was tested with this armature by comparing the readings taken with the armature almost tight in its bearings, and the magnet running very smoothly, with the readings taken when the armature was very loose. This has the effect of causing excessive vibration, the armature rattling to and fro at every revolution. As might be expected, the hysteresis is diminished (Diagram 7), but not by much, and it may therefore be concluded that the small vibration present, under normal working, does not influence the values to any appreciable extent.

A similar experience of the comparatively small effect produced by violent mechanical vibration is mentioned by Evershed and Vignolles ('Electrician,' 15th and 22nd May, 1891).

The effect of passing from a high induction to a low one, allowing a short time between each reading, was examined, the result being shown in Diagram 8. There was no definite change in the hysteresis, the curves of rising and falling currents being practically identical. This point will be referred to below, when more rapid changes are taken.

The effect of speed on hysteresis will be examined later, when experiments will be described in which the induction is kept constant and the speed varied.

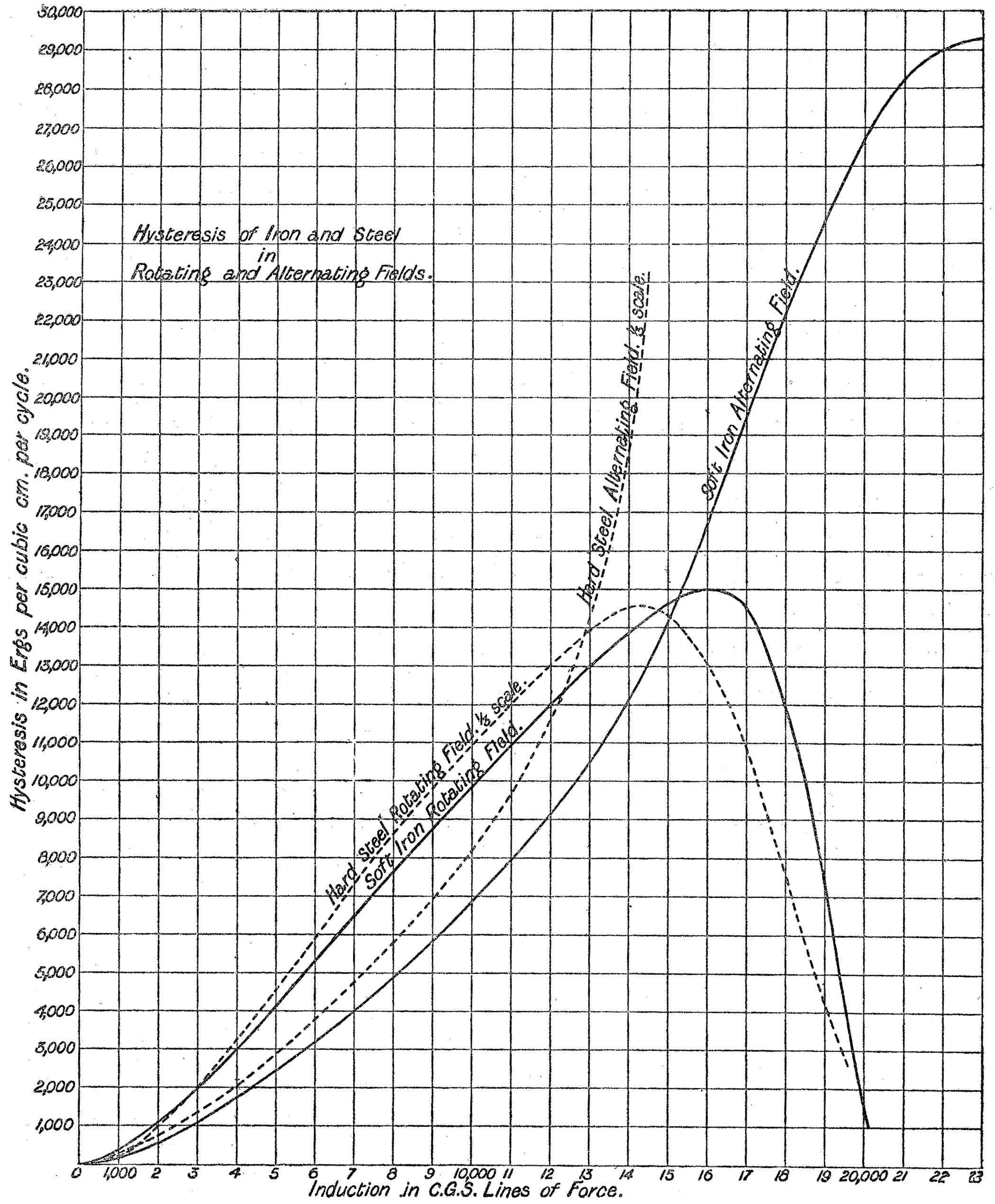
Finally, a test was made at an indefinitely slow speed by slowly turning the armature through a short distance and then stopping it. The deflexion was then read, while it was standing still. Since the smallest error of centring produced an additional pull either forwards or backwards, there are some irregularities in the readings, and a high induction could not be reached. The result is in complete accordance with the test at higher speeds, but it will be noted that the maximum value, though occurring at the same induction, viz., between 16,000 and 17,000, is not so high as before. This is due to a small backward movement of the armature immediately the magnet stopped, perhaps a kind of readjustment, which does not have time to take place when running. It is only a small effect, about 10 per cent. of the value of the hysteresis at that point, and is less marked at points beyond the maximum. The smallest actual rotation was sufficient to prevent its occurrence, the values then being closely in accordance with those at higher speeds. The curve is shown in Diagram 9.

[As it was found that the speed of rotation was without influence on the hysteresis, and that the eddy currents were almost negligible at moderate speeds, the greater number of readings were taken at speeds of 15 to 30 revolutions per second, since the deflexions were very much steadier than at low speeds. With the latter there was a tendency for the spot of light to oscillate, especially at high inductions, owing to small errors in centring, which it was almost impossible to eliminate altogether. The results obtained at low speeds, down to a speed of one revolution per second, were identical with those at high speeds.—2nd September, 1896.]

The above experiments prove very clearly the nature of hysteresis in a rotating field, and bear out most completely Mr. SWINBURNE'S deduction from the molecular theory of magnetism. The agreement of the phenomena with the previously suggested deduction forms a strong verification of the truth of this theory while it also easily explains various other points which have been noted. The three stages of the curve correspond precisely with the three stages of the molecular movement, and the difference between the curves for iron and steel readily follow from the difference in their structure. In the soft iron, as has been shown by EWING, the molecules move in larger combinations than in steel, and hence it is not until a higher induction is reached that uniformity of movement is produced. When once past the maximum the decrease is very rapid, and both metals appear to give curves reaching approximately a zero value at an induction between 20,000 and 21,000.

It is an important part of Professor EWING's theory of magnetism that hysteresis is caused solely by intermolecular magnetic control, and not by any friction on the moving molecule, although there are great temptations to assume some such friction in certain phenomena. But the almost complete disappearance of hysteresis in the soft iron proves that friction must be almost entirely absent, since an individual rotary motion would not avoid this.

Diagram 10.



The curve for hard steel is not so conclusive on this point, as the induction has not been pushed far enough, but so far there is no sign of any large amount of frictional control. This is noteworthy, as we are accustomed to consider the steel molecules as subject to some form of mechanical constriction.

In Diagram 10 are given the mean values of the curves obtained for iron and steel, the latter being plotted with a smaller vertical scale to admit of easier comparison. With them are also plotted the curves of hysteresis in an alternating field, that for soft iron being obtained by a calorimetric method with iron from the same sample. ('Electrician,' November 22, 1895.)

PART II.

Effect of Speed on Hysteresis.

The question of the relation between the speed of reversal of magnetization and the value of the hysteresis per reversal has been investigated by several experimentalists, by the employment of an alternating current of varying periodicity.

The experiments of TANAKADATÉ ('Phil. Mag.,' September, 1889) by the calorimetric method, indicate that there is no change in the hysteresis between the values 27 and 400 cycles per second. His method, however, is liable to error, and he does not measure the induction at the same time as the hysteresis, but finds it by a ballistic test. As his magnetic circuit has only a small reluctance and no air gap, this assumption of identity is doubtful.

The experiments of Mr. A. SIEMENS ('Proc. I.E.E.,' February, 1892) and Mr. C. P. STEINMETZ ('E.T.Z.,' 1891 and 1892, and 'Electrician,' February, 1892) indicate a change of hysteresis in this respect, but their results are not consistent. By the magnetic curve tracer Professor EWING has made some tests at low speeds, but the presence of eddy currents in the pole pieces obscure the results, and mechanical lag and vibration in the moving parts prevent the use of any but very moderate speeds. Dr. JOHN HOPKINSON ('Electrician,' *loc. cit.*) finds that in hard steel wires, while the permeability is not changed, there is a small increase in the hysteresis as the speed is increased. The experiment is, however, obscured in regard to this point by the changes in the form of the current curve consequent upon change of speed. Mr. T. GRAY ('Roy. Soc. Proc.,' May, 1894) finds no variation in the hysteresis between the speeds of 3 and 8,000 cycles per minute, but his experiments are somewhat contradictory. Recently the author has shown ('Electrician,' *loc. cit.*) that the change of hysteresis, if it exists at all, must be very slight and that on theoretical grounds it is probably independent of speed.

In the foregoing experiments with a rotating field it has been indicated that there is at most very little change of hysteresis at different speeds. The point was more carefully worked out as follows :—

The machine was run at a gradually increasing speed, and the current was kept perfectly constant. As has been pointed out, this ensures a practically perfect constancy in the value of B in the armature.

At first with the steel armature, a distinct variation was found, the sign of the change depending on the induction; but the effect was traced to errors in the truth of the armature. When these were eliminated the result of a large number of tests is to show that up to a speed of 70 revolutions per second there is no regular or definite change in the hysteresis when tested in the rotating magnet machine.

All the experiments were performed by keeping the current constant and unbroken, and increasing the speed step by step, waiting at each reading until the reading had become steady.

The tests of the steel are shown in Diagram 11. There are irregularities which could not be eliminated, and which frequently repeated themselves at the same points, but on the whole there is a clear indication of no sensible change. The readings are corrected for eddy currents.

The values at the maximum, or near to it, tend to be irregular, as was seen also in the previous curves. This may well be expected, the metal being to some extent in a critical condition.

The values at the highest speeds were rendered slightly uncertain owing to vibration, which shook the spot of light, but there is scarcely a definite change in one direction more than the other.

The soft iron shows the same kind of curves, and the same kind of small variations, but it may be noted that the readings given in Tables XI. to XIV. are not so regular as in the case of hard steel, and repetition of the series does not reproduce the same irregularities.

At the value B = 17,200, four series were taken at different times; two of them show slightly rising values, another falling values, while a fourth gives an approximately constant value. An example of a rising curve is shown at B = 7,800, but on repetition this gave a practically horizontal line.

On repeating the tests with a smaller armature, the same results appear. At the maximum value of the hysteresis when B = 16,500, the readings were very irregular, as was anticipated, since the iron was in a critical condition. The same effect is noticed to a lesser degree near the maximum, when B = 15,000 and 17,200.

A large number of series have been given because the actual readings show irregularities which might prevent a generalization; but after constant repetition it is seen that the variations obey no regular law, and are probably due partially to errors in the machine, though to some extent there is strong ground for concluding that they are due to actual variations in the value of the hysteresis.

The method is more accurate than those which entail the accumulating of the waste of energy, since in them the amount measured is proportional to the speed, and the conditions are changed. The deflections here are quite unaffected by the speed, and depend only on the value of the hysteresis at that moment. Hence the small evanescent variations may be readily detected, whereas by the cumulative methods they would be merged in the general average.

There are a few points concerning the effect of time and sudden changes which are interesting. As has been mentioned, the curves were taken with steadily increasing current or speed, a short time being allowed before a reading was taken.

When, however, a sudden increase is made in the current, the value of the hysteresis is higher than the normal for that current and the value slowly decreases until it reaches the normal steady condition. The effect is not large, about 3 or 4 per cent., but is by no means regular. The time of its disappearance is usually a few seconds and the effect is greater at medium and high induction than at low induction. The reverse effect is shown in suddenly decreasing the current, the hysteresis having too low a value, and not rising up to its normal for upwards of a minute, the arrival at the normal being slower than with a rising current.

A similar effect is observed in changes of speed. A rapid rise in speed gives an increased value of the hysteresis, which dies away in a few seconds. But a rapid reduction in speed gives a diminished value of the hysteresis which lasts often more than a minute, during which time the deflexion is very irregular, jumping up to the normal and going back again. The change is also greater than any of the preceding ones, and is frequently as much as 10–15 per cent.

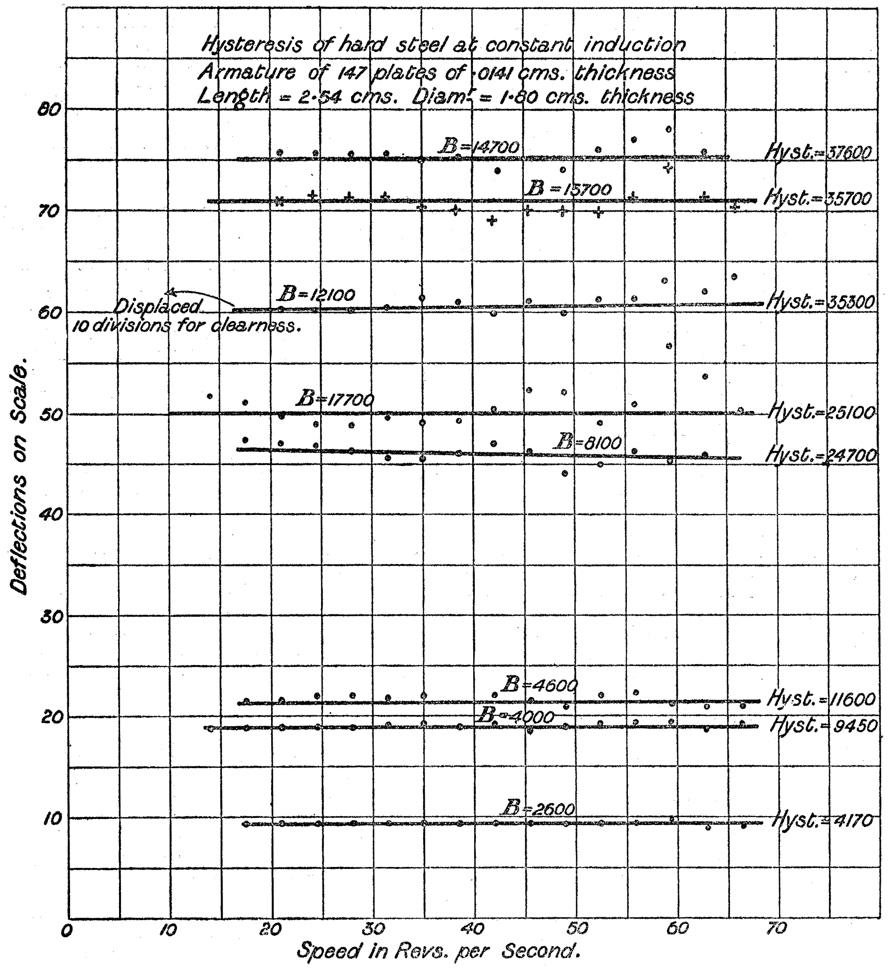
The effect of rapid makes and breaks in the magnetising current in quick succession is to increase the hysteresis. It recovers its normal value after a time but the readings are very irregular during the recovery. In all these effects, the machine is kept constantly running, and they occur more or less at all speeds. They are, however, not regular enough in amount to allow of any connection with the speed being established.

Effects somewhat similar to these have been noticed with alternating currents. Tomlinson mentions ('Proc. Roy. Soc.,' Dec. 1889), that repetition of the cycle reduces the hysteresis, and he names it the "accommodation" of the molecules, supposing that the molecules become arranged in a manner in which it is more easy for them to reverse, and it is not surprising that with the more uniform motion of a rotating field, the same effect should be observed. The effect of suddenly diminishing the speed is more obscure; but in these experiments it seems to be due to a disarrangement and an absence of recombination for a short time, giving rise to great irregularity of hysteresis. The length of time over which the effect lasts is remarkable, as it often extended to over 1,000 cycles. It was completely stopped by a temporary stoppage of the current, while Tomlinson found that even slight mechanical shocks were sufficient to stop the effect of "accommodation."

The effect of reducing the current is probably due to the same cause. The molecules are arranged in regular order, and it is some time before they disarrange themselves, while during the process the hysteresis is lower than the normal. An increase in the current is not immediately followed by complete arrangement of the molecules, and during the process the hysteresis is too high. On breaking and remaking the circuit after a sudden reduction in the current, the deflexion of the armature, previously lower than the normal, rises to a value higher than the normal and then drops to its normal value, showing that by breaking the circuit, the regular arrangement is completely disturbed.

Professor EWING in his book ("Magnetism in Iron," &c.) mentions that rapid makes and breaks increase the permeability, but increase still more the residual magnetism,

Diagram 11.



but he does not give the hysteresis effect, which may or may not have been increased. This is clearly a case of "accommodation." When, however, the current is reversed, the increased amount of combinations renders the iron less permeable with more "coercive force" necessary, and therefore with presumably more hysteresis. The result does not seem to agree very well with that of TOMLINSON'S.

In the rotating field it is possible that rapid makes and breaks tend to disarrange the symmetrical arrangement produced by continued rotation, and to increase the number of combinations. This would account for the increase in the hysteresis, and also for its irregular value before it again reached its normal value.

[It will be noted that while the average values of hysteresis obtained in the Tables X. to XIV. agree for the most part very closely with those in the previous tables; the values at the maximum are distinctly lower, both in iron and steel, indicating a greater "accommodation" under the long continued running in the critical condition.—2nd September, 1896.]

In the hard steel these effects are very much diminished, the deflexions being more uniform and consistent. The same difference is shown in the constancy of the hysteresis at the maximum point. In the steel, all the curves agree very closely in the maximum value attained, Diagram 4, and the speed curve at that point, Diagram 11, although somewhat irregular, is much less so than the curve from soft iron. This follows from the supposition that in steel the aggregations of molecules are smaller and less dependent on each other, and hence the average behaviour is more regular.

Table X.—Hard Steel Armature of 147 Plates, of thickness ·0141 centim. Variation of Hysteresis with Speed; Diagram 11; Deflexions on Scale: 1 division = 502 ergs per cub. centim. per rev.

Speed in revs. per sec.	B=2600.	4000.	4600.	8100.	12,100.	14,700.	15,700.	17,700.
·14	9·3	18·8	..	..	..	..	..	51·6
17·5	9·3	18·8	21·5	47·5	..	..	..	51·0
21	9·3	18·8	21·5	47·0	70·2	75·6	71·0	49·9
24·5	9·3	18·8	22·0	46·7	70·1	75·5	71·5	48·9
28	9·3	18·8	22·0	46·3	70·1	75·4	71·4	48·8
31·5	9·3	18·8	21·7	45·7	70·5	75·4	71·4	49·7
35	9·3	19·0	22·0	45·5	71·5	75·1	70·3	49·1
38·5	9·3	18·8	..	46·0	71·0	75·4	70·2	49·5
42	9·3	19·0	22·0	47·0	69·8	73·8	69·1	50·4
45·5	9·3	18·7	21·6	46·0	71·2	75·2	70·2	52·3
49	9·3	18·8	21·0	44·0	71·3	74·1	70·2	52·1
52·5	9·3	19·0	22·0	45·0	..	76·2	70·0	49·1
56	9·3	19·1	22·3	46·0	71·3	77·3	71·4	51·0
59·5	9·5	19·1	21·3	45·5	73·2	78·4	74·8	56·9
63	8·9	18·5	21·0	45·5	72·2	75·9	71·7	53·8
66·5	9·0	19·0	21·0	..	73·6	..	70·6	50·2
Value of eddies in scale division	·0005n	·0013n	·0016	·0050	·012n	·017n	·020n	·025n
Average value of hysteresis	4170	9450	11,600*	24,700*	35,300	37,600	35,100	25,100

* Value of spring 1 division = 537 ergs per cub. centim. per division.

TABLE XI.—Soft Iron Armature of 250 Plates, of thickness ·0081 centim. Variation of Hysteresis with Speed.

Speed in revs. per sec.	Deflexion.	Speed in revs. per sec.	Deflexion.
7·5	109	7	69
10	108	10	69
23	109	14	67·5
27	109·5	17	68
31·5	110·5	21	67·5
37	112	23	70·5
43	111·5	26·5	70
52·5	111	34	71
61	110	40	71
71	107·5	45·5	71
B = 4800 Hysteresis = 3500 Eddy currents $= 1{\cdot}2 \times 10^{-2}\, n$ divisions 1 scale division = 32 ergs per cub. centim. per rev.		51	71·5
		57	71
		62	69
		65	67·5
		70·5	69
		B = 3400 Hysteresis = 2200 Eddy currents $= 6 \times 10^{-3}\, n$ divisions	

TABLE XII.—Soft Iron Armature of 250 Plates, of thickness ·0081 centim. Variation of Hysteresis with Speed.

Speed in revs. per sec.	Deflexion.	Speed.	Deflexion.	Speed.	Deflexion.	Speed.	Deflexion.
						Repeated.*	Repeated.*
0	8·0	0	35·5	15	41·2	0	31·5
23·5	8·5	25	35·5	18·5	40·7	29	32·0
30	8·5	32	35·6	23	40·1	32	32·4
36	8·5	38	35·6	27	38·6	40	31·8
43	9·0	44·5	35·5	30	38·1	42	31·3
50	8·5	51	36·1	34	38·2	45	31·2
57	8·5	57·5	36·0	38	37·6	54	32·1
62	9·0	64	35·9	41	37·5	59	31·5
B = 4800 Hysteresis = 3800 Eddy currents $= 1{\cdot}1 \times 10^{-3}\, n$ divisions		B = 14,000 Hysteresis = 14,000 Eddy currents $= 9{\cdot}0 \times 10^{-3}\, n$ divisions		49	38·0	62	31·4
1 division = 385 ergs per cub. centim. per rev.				57·5	38·2	67	31·4
				62	38·8	B = 17,100	
				66·5	38·2		
				B = 17,100 Hysteresis = 14,700 Eddies $= 1{\cdot}3 \times 10^{-2}\, n$ divisions			

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* This series was repeated later, and the spring was not calibrated. It had evidently changed in strength or method of applying restoring force.

TABLE XIII.—Soft Iron Armature of 250 Plates, of thickness ·0081 centim. Variation of Hysteresis with Speed; Deflexions.

Speed in revs. per sec.	B=2000.	2750	4400	7800	11,400	15,000	17,200	17,200
2	..	4·0	..	..	31·5			
7	..	4·0	7·8	18·2	31·4	37·6	38·8	39·1
14	2·8	4·1	7·9	18·3	31·4	38·2	38·7	39·3
17·5	2·6	4·2	7·9	..	..	..	38·8	38·8
21	2·6	4·2	8·1	18·7	32·1	38·6	39·1	38·1
24·5	2·4	4·3	8·1	19·0	31·8	39·1	39·5	38·1
28	2·6	4·4	8·2	19·1	31·2	38·7	40·1	40·1
31·5	2·6	4·4	..	19·3	31·4	37·9	40·1	39·0
35	2·7	4·4	8·2	19·2	31·5	38·1	39·7	40·6
38·5	2·7	..	..	19·6	31·8	38·8	40·2	39·9
42	2·7	4·3	8·2	19·4	30·8	37·9	39·5	38·6
45·5	2·7	4·3	8·5	19·7	30·8	38·4	39·9	40·2
49	..	4·4	8·4	20·3	30·5	37·6	39·1	41·2
52·5	..	4·2	..	20·5	30·7	38·4	39·1	39·8
56	..	..	..	20·5	31·5	38·9	40·2	40·2
59·5	..	..	..	21·2	31·9	40·4	41·2	41·6
63	..	..	..	20·6	32·2	39·9	41·7	41·2
66·5	..	..	..	..	32·6	..	43·6	
Hyst. ..	1050	1530	3060	7660	12,100	14,500	14,700	14,700
Eddy currents (div.)		·001n	·002n	·003n	·006n	·010n	·014n	·014n

TABLE XIV.—Soft Iron Armature of 156 Plates, of thickness ·0031 centim. Variation of Hysteresis with Speed; Diagram 17; Deflexions: 1 division = 775 ergs per cub. centim. per revolution.

Speed.	B = 5150	11,000	16,500	18,600
14	..	..	..	16·9
17·5	5·5			
21	5·5	13·9	16·1	16·9
24·5	5·5	13·9	16·6	
28	5·6	14·4	18·0	17·3
31·5	5·6	14·7	18·0	
35	5·6	14·9	16·0	16·8
38·5	5·6	14·8		
42	5·3	14·6	18·4	16·2
45·5	5·2	14·6	19·3	
49	5·3	14·8	18·9	16·7
52·5	5·7	14·7	19·9	
56	6·0	14·4	18·8	17·1
59·5	6·2	14·7	17·8	
63	5·9	15·2	16·9	17·6
66·5	5·8	..	17·3	17·0
Value of Eddy currents in scale divisions . . .	$= \cdot 0006n$	$= \cdot 003n$	$= \cdot 006n$	$= \cdot 007n$
Hysteresis mean value in ergs . .	4350	11,200	13,700	13,000

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